Virtual Images and Normal Moveout

Vigen Ohanian, Texaco Inc.

SUMMARY

Building on the foundation laid by de Bazelaire (1988), and Castle (1988), I further examine the use of the method of virtual images for generating zero offset sections.

In this paper, I first derive Castle's expression for the two parameter NMO equation by a strict application of the physical conditions for the formation of a virtual image. The physical approach taken here, circumvents the use of complicated mathematics and the need to resort to Gauss' Hypergeometric series. As a result of the present treatment of this problem, I was able to demonstrate that this is the most general NMO representation derivable from these physical conditions. Expressions for the image depth and the imaging velocity are additional byproducts of this approach.

After investigating various forms of the two parameter NMO equation, the special case of the one parameter equation is derived. Subsequently, the merits of these NMO equations are analyzed for different geologies. The analysis confirms de Bazelaire's synthetic results. The one parameter NMO equation is ineffective for imaging flat layers, however, it can be useful in imaging diffractions and flat reflectors below dome-like structures. The two parameter equation proved effective in all cases. Some of the conclusions are borne out in real data examples.

CONDITIONS FOR FORMATION OF VIRTUAL IMAGES

When we observe a swimming pool, it appears shallower than its true depth. Light rays emerging from the bottom of the pool, enter the air and refract away from the vertical. Rays traveling near the vertical converge at the same point, called an image point. To the observer, these rays appear to emanate from the image point, which occurs above the actual depth of the pool. The convergence of rays at the image point and their simultaneous arrival from the image point to the observation point secures the formation of the image. Since the refracted rays do not actually go through the image point, such an image is called a virtual image.

When dealing with reflection seismology, the same two conditions are required for the formation of an image. First, to insure the existence of an image, all rays emerging from the reflection point must converge to the same point. Second, to insure the formation of the image, these rays must arrive from the image point to all observation points, simultaneously. Waves arriving in step, contribute to a concentration of energy at the image point. Here, lies the practical significance of the imaging technique. In practice it is possible to determine the correct imaging times through a search for the largest stack energy.

NMO BY THE FORMATION OF VIRTUAL IMAGES

Figure 1 shows the ray path for a reflection in a horizontally layered earth. The shot is at S, the reflection is from the Nth layer, and the receiver is at R. The shot to receiver offset is X. For the purposes of reflection angles and arrival times, this model is equivalent to the one shown in Fig. 2, where all layer thicknesses are doubled and only upcoming waves are considered. The source is now buried at the lowest depth, and the ray path from the source to the receiver R, in respective layers, is along the same angles as in Fig. 1.

For the ray path indicated on figures 1 and 2, Snell's law states that.

\[
\sin \theta_k = V_k P, \tag{1}
\]

where, \( V_k \) is the wave velocity of the k-th layer and \( \theta_k \) is the ray angle from the vertical. and \( P \) is the ray parameter.

Now according to Eq. (21) of Castle, the ray parameter can be expressed in terms of the offset:

\[
P = \sum_{j=0}^{\infty} \frac{1}{j!} \frac{1}{x^2} \tag{2}
\]

where, the coefficients \( \Gamma_j \), are given by Castle's Eq. (22). The first three values of \( \Gamma_j \) are:

\[
\Gamma_0 = \frac{1}{t_0 m_2}, \quad \Gamma_1 = -\frac{m_4}{8t_0^2 m_2^2}, \quad \Gamma_2 = -\frac{3m_6}{4t_0^4 m_2^4} \tag{3}
\]

where, \( t_0 \) is the two way vertical travel time to the \( N \)th layer, and \( m_j \) are the time weighted moments of the velocity distribution.

\[
\Gamma_j = \left( \sum_{i=1}^{N} t_{ik} V_i \right) \left( \sum_{i=1}^{N} t_{ik} \right) \tag{4}
\]

To handle the problem in the most general way, the multilayered model of Fig. 2 is replaced with a uniform velocity medium, which permits the formation of a virtual image to the highest order of approximation in the offset. To obtain the correct velocity, for the equivalent uniform medium, consider the ray shown in Fig. 2 and its ray parameter expansion given in Eq. (2). Now a ray in the equivalent uniform medium of velocity \( U \), having the same ray parameter \( P \), will emerge at \( x \) at an angle \( \theta \) given by:

\[
\sin \theta = UT_{0} x + UT_{1} x^2 + UT_{2} x^3 + UT_{3} x^4 + \ldots \tag{5}
\]

From the simple geometry in Fig. 3, we see that the virtual ray, connecting an image point at \( z \) to the observation point at \( x \), defines \( \tan \theta = x/z \). Substituting this into (5) to eliminate \( \theta \), and using a Taylor series expansion in \( x \), we have:

\[
z = \frac{1}{UT_{0}} \left( \frac{UT_{1} x^2}{2} + \frac{UT_{2} x^4}{4} \right) + \ldots \tag{6}
\]

Now, to insure the existence of a virtual image, \( z \) must be independent of the offset \( x \). For small offsets, in the above expansion only the first term will contribute. Thus, when

\[
\left( \frac{x^2}{2t_{0}^2 m_2} - \frac{U^2}{m_2} \right) < 1
\]

for any choice of \( U \), rays will converge to the same depth point \( z = t_0 m_2 / U \). For the particular choice of \( U = V_i \), to the lowest order in offset, the depth of the virtual image is:

\[
z_v = \frac{1}{V_i} m_2 \tag{7}
\]
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To go from the image depth to the focusing time \( T_o \), both sides are divided with \( V_i \):

\[
\frac{T_o}{t_o} = \frac{m_2}{V_i^2}.
\]

(9)

This agrees with de Bazelaire's result of P. 149.

For a higher order approximation, we set the coefficient of \( x^2 \) term in (6) equal to zero. This gives the equivalent medium velocity to be:

\[
U^2 = \frac{m_3}{m_2}.
\]

(10)

Thus, when

\[
\left( \frac{3x^4}{8t_0^2 m_2^2} \right) (m_2^2 - m_3 m_4) << 1
\]

(11)

rays will focus at the same depth point:

\[
z^2 = \frac{r_0^2 m_2}{m_4}.
\]

(12)

In as much as the substitution of Eq. (10) into Eq. (6) eliminates the \( x^2 \) term of the expansion, it however, does not eliminate the \( x^4 \) term. Thus, Eq. (12) gives the most general expression for the depth of the virtual image in a multi-layered media. That is, the expression is valid to the highest order in offset, allowed by the physical process of image formations. By the same token, the velocity given by (10) is the most general expression for the velocity by which a virtual image can be secured in a flat multi-layered media.

In passing we note that, after the substitution of Eq. (10) into (6) the coefficient of the \( x^4 \) term in the expansion is found to be proportional to \((m_2^2 - m_3 m_4)\). Hence, for media with small accelerations this quantity will be small and the fourth order term in (6) will vanish as well. As a result, for most geologies, the above general statements will remain valid.

This concludes the discussion of the condition for the existence of a virtual image. Now, to insure the formation of an image, it is necessary for the rays to arrive simultaneously, from the image point to the various offsets (observation points). Since rays along the diagonal travel a longer distance than the vertical ray, clearly, they will be delayed. Hence, a moveout is necessary.

Let the difference between the actual travel times \( t(x) \) and the virtual travel times \( T(x) \) be \( t_r \):

\[
T(x) - t(x) = t_r.
\]

(13)

To secure an image, it is sufficient for \( t_r \) to be offset independent, that is, \( t_r \) must be a constant. An implication of this condition is:

\[
T(x) - T(0) = t(x) - t(0).
\]

(14)

That is, when the actual reflections are NMO corrected, and hence are made to arrive simultaneously at \( t_o \), then the virtual rays will also arrive in step; but will be delayed from \( t_o \) by the amount \( t_r \). Now, referring to Fig. 3, and utilizing the statement in (14), for the right angle triangle we write:

\[
\left[ U(t(x) - t_o) + z \right]^2 = z^2 + x^2.
\]

(15)

Substituting (10) and (12) into (15) and solving for \( t \) we have:

\[
(t(x) + t_r)^2 = \left( t_o + t_r \right)^2 + \left( \frac{x}{U} \right)^2.
\]

(17)

This is the equation of a hyperbola whose center has a shift of \(-t_r\). It is de Bazelaire's generalized two parameter equation. The unknown parameters are the shift \( t_r \) and the unknown velocity \( U \). When \( U^2 = m_1^2/m_2 \), and \( t_r = t_r(m_2^2/m_1 - 1) \), then Eq. (17) gives Castle's result (16). When \( \frac{m_2}{V_i} \), Eq. (17) has only one unknown parameter, namely the shift \( t_r \). In this case, with the use of Eq. (13), the expression in (17) reduces to de Bazelaire's one parameter equation:

\[
\left[ (t(x) - t_o) - T_o \right]^2 = T_o^2 + \left( \frac{x}{V_i} \right)^2.
\]

(18)

COMPARISON OF THE NMO EQUATIONS

A comparison of the performance of the NMO equations (16) and (18) with the conventional NMO equation for imaging (a) flat layers, (b) a flat reflector below a dome, and (c) diffractions, is now given.

a) Imaging Flat Layers

A systematic comparison of the two parameter equation (16) with the conventional NMO was given by Castle (1988). A similar approach is now taken, to compare the one parameter equation with the conventional NMO equation. Using Eq. (3), the one parameter equation and the conventional NMO equation are first expressed in terms of the \( \Gamma \) coefficients, and then are expanded in a Taylor series in \( x \). The expansions are:

\[
t_{par} = t_o + \frac{\Gamma_0}{2} x^2 - \frac{\Gamma_0^2 V_i^2}{8} x^4 + \ldots HO \ldots,
\]

(19)

\[
t_{con} = t_o + \frac{\Gamma_0}{2} x^2 - \frac{\Gamma_0^2}{8t_o} x^4 + \ldots HO \ldots
\]

(20)

Now, according to Castle's Eq. (27) the exact NMO equation for the horizontally layered earth model is:

\[
t_{exact} = t_o + \frac{\Gamma_0}{2} x^2 + \frac{\Gamma_1 x^4}{4} + \ldots HO \ldots
\]

(21)
Let, $e_{\text{par}} = t_{\text{exact}} - t_{\text{par}}$ and $e_{\text{con}} = t_{\text{exact}} - t_{\text{con}}$, define the errors in the one parameter and the conventional NMO equations. Then, the difference of the squares of these errors, is:

$$\left( e_{\text{par}} \right)^2 - \left( e_{\text{con}} \right)^2 = \left( \frac{m_z}{8t_c m_x^2} \right)^2 \left( V_t^2 - m_x \left( \frac{m_x}{m_y} (V_t^2 + m_y) - 2 \right) m_x \right) m_x. \quad (22)$$

For $V_t^2 < m_x$, which is usually the case, the above expression is positive. Hence, for usual velocity profiles and usual choices for $V_t$, the conventional NMO equation will image flat layers better than the one parameter equation.

b) Imaging a Flat Reflector Below a Dome:

Consider the model of a flat reflector below a spherical dome shown in Fig. 4. To simplify things let the reflector pass through the origin of the sphere. Furthermore the shot and the receiver are placed symmetrically about the center of the sphere. For the depicted ray path, we have:

$$x^2 + z^2 = \left( t(x) - t_0 \right) V_t^2, \quad (23)$$

which is exactly in the same form as the one parameter NMO equation (15). Dividing both sides by $V_t^2$ and substituting $T_0$ for $z/V_t$, gives the Eq. (18). Thus this particular reflection point can be exactly imaged by the one parameter equation.

The present analysis pertains to a very special set up of the geology and the acquisition geometry. However, reasonable deviations from this model should not change things significantly. This is evident in figures 8 and 9 of de Bazelaire. All this demonstrates the effectiveness of the one parameter NMO equation for imaging structures below domes.

c) Imaging Diffractions:

Figure 5 displays a diffraction process in which the diffractor is embedded in a layered medium. The diffraction travel time, for the ray shown in the figure, is computed by using the Eq. (21) twice: Once, for the downgoing wave (to the diffractor), and next, for the upcoming wave (to the receiver). Adding the two times gives the following exact expression for this diffraction process:

$$t(x) = t_0 - \frac{\pi}{2} \sum_{k=1}^{n} \frac{1}{k} x^k + \frac{1}{4} \sum_{k=1}^{n} \frac{1}{k} (2k-2) (R \cos \phi)^{(2k-2)} x^k,$$

$$+ \frac{1}{16} \sum_{k=1}^{n} \frac{1}{k} x^k + \frac{1}{4} (R \cos \phi)^{(2k-4)} x^k + ... HO... \quad (24)$$

where, the binominal coefficients are defined in the usual way. In Eq. (24), $R$ is the distance from the midpoint to the diffractor, and $\phi$ is measured from the horizontal. For the special case of $\phi = 90^\circ$, the expression in (24) reduces to Eq. (21). Thus, the apex of a diffraction pattern is best imaged by the two parameter new NMO equation. The conventional NMO and the one parameter equation will produce poorer results respectively.

Another special case of (24) involves a diffractor in a uniform medium, which includes the physical process of water-bottom diffractions. For this case, the square of the travel time is given by:

$$t(x)^2 = t_0^2 + \left( \frac{\sin \phi}{v} \right)^2 x^2 + + \left( \frac{\sin^2(2\phi)}{4V^2 t_c^2} \right) x^4 + ... HO... \quad (25)$$

where, $t_0 = 2R/V$. In Eq (25), it was also assumed that $R^2 >> x^2$.

Once again, for a diffractor below the midpoint Eq. (25) reduces to the conventional NMO equation. Since for this particular geometry $T_0 = t_0$, then the one parameter equation will also reduce to this same result. Thus, for a uniform medium, the apex of the diffractor is imaged equally well by the one parameter and the conventional NMO equations.

However, when $\phi$ is different from $90^\circ$, the fourth order term in (25) is positive. Upon squaring Eq. (19) it can be shown that, the fourth order term in the one parameter equation is also positive. Thus, broadly speaking it can be concluded that the flanks of deep water bottom diffractions will be better imaged by the one parameter NMO equation than by the conventional NMO equation.

DATA EXAMPLES

Figures 6 (a) and 6 (b) show the results from processing land data, using the conventional and the one parameter NMO equations. The data has a layer cake geology. It is seen that the conventional processing has produced a better stack of the flat layers. Figures 7(a) and 7(b) are results from deep marine data. Diffraction hyperbolas appear to be imaged better in the one parameter result than the conventional processing.

CONCLUSIONS

The treatment of the NMO problem by the method of virtual images given here, allows two general conclusions: First, that Castle's representation of the two parameter NMO equation is derivable by a strict application of the physical conditions for the formation of a virtual image. Second, that this is the most general NMO expression attainable via the stated physical conditions.

Expressions for the image depth and the imaging velocity were additional byproducts of the approach taken here. The one parameter equation was shown to be a lower order approximation in offset than the two parameter equation. The one parameter equation has a smaller aperture than the two parameter equation.

The NMO equations were compared for imaging flat layers, a flat reflector below a dome, and diffractions. The conclusions confirmed de Bazelaire's synthetic results. The one parameter equation is ineffective for imaging flat layers, however, it can be useful for imaging reflections from below dome-like structures. To handle diffractions, an exact expression for diffraction travel times in a multi-layered media was derived. One special case of the equation showed that, deep water bottom diffractions are better imaged by the one parameter equation than the conventional NMO equation. The two parameter equation proved effective in all cases.

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Fig. 1 Layered Earth Model

Fig. 2 Equivalent Model

Fig. 3 Virtual Ray Diagram

Fig. 4 Flat Reflector Below a Dome

Fig. 5 Diffraction Ray Path

Fig. 6a One Parameter NMO

Fig. 6b Conventional NMO

Fig. 7a One Parameter NMO

Fig. 7b Conventional NMO